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Publication date:
2011

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Citation for published version (APA):

Lindelauf, R., Hamers, H. J. M., & Husslage, B. G. M. (2011). *Game Theoretic Centrality Analysis of Terrorist Networks: The Cases of Jemaah Islamiyah and Al Qaeda*. (CentER Discussion Paper; Vol. 2011-107). Operations research.

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No. 2011-107

**GAME THEORETIC CENTRALITY ANALYSIS OF TERRORIST
NETWORKS: THE CASES OF JEMAAH ISLAMIYAH AND
AL QAEDA**

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September 28, 2011

ISSN 0924-7815

Game theoretic centrality analysis of terrorist networks: the cases of Jemaah Islamiyah and Al Qaeda.

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Abstract

The identification of key players in a terrorist network can lead to prevention of attacks, due to efficient allocation of surveillance means or isolation of key players in order to destabilize the network. In this paper we introduce a game theoretic approach to identify key players in terrorist networks. The advantage of this approach is that both the structure of the terrorist network, which usually reflects a communication structure, as well non-network features, which represent individual parameters like financial means or bomb building skills, can be taken into account. The application of our methodology results in rankings of the terrorists in the network. We illustrate our methodology by two case studies: Jemaah Islamiyah's Bali bombing and Al Qaeda's 9/11 attack, which has led to new insights in the operational networks responsible for these attacks.

Keywords: terrorism; network analysis; centrality; game theory.

JEL classification: C71.

1 Introduction

A major problem in counterterrorism practice is determining which individuals are the key players in a terrorist organization. The current practice of key leader engagement is often based on qualitative theories, such as theories of charismatic leadership (Jordan (2009)). With the huge increase in digital information gathering, intelligence and law enforcement agencies possess large volumes of raw, heterogeneous, often incomplete and inaccurate data on terrorist networks (McAndrew (1999), Sparrow (1991)). The use of sophisticated quantitative modeling techniques and procedures to clean and make sense of these data is however limited (Xu and Chen (2005)). One of the quantitative methodologies that is often applied to find the proverbial needle in the haystack in general social networks is *social network analysis* (Wasserman and Faust (1994)). This methodology has also been applied to terrorist networks, see, e.g., Koschade (2006). A common feature of social network analysis is that it only uses the structure of a network. In this paper we introduce a methodology that incorporates, besides the network structure, additional information available on a terrorist group in the analysis of the social network underlying the

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terrorist group. We show that quantitative modeling by means of cooperative game theoretic centrality measures enables the incorporation of such additional information.

Several researchers have shown how complex data on criminal organizations can be analyzed using the *network perspective*, e.g., Sparrow (1991), Peterson (1994) and Klerks (2001). Quantitative analyses of terrorist networks include Carley et al. (2003), Farley (2003) and Lindelauf et al. (2009).

The strength of social network analysis lies in the fact that one takes interrelationships into account when analyzing a group of people (Ressler (2006)). Centrality analyses can be applied to find the most important person in a social network. Clearly, ‘most important’ depends on the context of the problem under consideration, hence, many different centrality measures have been developed. A centrality analysis leads to a ranking of individuals that are active in the social network. Three of the most well-known centrality measures arising in social network analysis are degree centrality, betweenness centrality and closeness centrality (cf. Wasserman and Faust (1994)). In this paper we refer to these three centrality measures as *standard centrality*. Software implementations of standard centrality are found in, for example, Ucinet (Analytic Technologies (2010)). Furthermore, Analyst’s Notebook (I2 (2010)), a software package used worldwide by law enforcement and intelligence agencies, has recently included standard centrality in its latest update. Unfortunately, most, if not all, centrality measures currently in use in the intelligence and law enforcement domain focus specifically on social network structure (that is, who communicates with whom), and do not incorporate other information that often is available. In the context of terrorist networks such additional information can be twofold: either information on individual terrorists, like financial means, bomb building skills, attendance of individuals at certain meetings, signs of radicalization or presence at a terrorist training camp, or either information on relationships between terrorists, ranging from the frequency and duration of interaction between individuals to the quantities of weapons being transported. Standard centrality is not able to incorporate this kind of data.

In this paper we use cooperative game theory to develop rankings of individuals in terrorist networks based on both the structure of the terrorist network and additional information on the terrorists and their relationships. Game theory is a mathematical theory of strategic interaction between individuals (often called players). Game theory can roughly be divided into cooperative game theory and non-cooperative game theory. Since the publication of von Neumann and Morgenstern’s classic work (Neumann and Morgenstern (1944)) game theory has spawned many fields. Traditionally game theoretic models are used in the field of economics, i.e., models of bargaining and auctions (Kalai (1977), McMillan (1994)). Other fields where game theory is applied range from military arms races (Guth (1988)), social and political sciences (Downs (1957), Shubik (1981)) to biology (Maynard Smith (1982)). Cooperative game theory has been used in networks to investigate how power is allocated (cf. Jackson (2008), Gomez et al. (2003)). In this paper we apply cooperative game theory to terrorist networks, which include, in contrast to the networks considered by Jackson and Gomez et al., features that do not only depend on the network structure. Clearly, a terrorist organization can be considered as a social network as it consists of players working together to achieve a goal. A typical example being a group of insurgents trying to carry out attacks with improvised explosive devices. To successfully launch such an attack several tasks have to be conducted; i.e., finances have to be arranged, the bomb material has to be acquired, the bomb has to be built and reconnaissance has to be conducted at the potential attack site. Hence, a terrorist group needs to consist of individuals capable

of performing such tasks. Moreover, terrorist groups heavily rely on communication networks to accomplish acts of recruitment and planning (Tsvetovat and Carley (2005)). Therefore, terrorist organizations can also be described by a social network. However, the structure of a terrorist network differs significantly from a general social network (cf. Lindelauf et al. (2009), Lindelauf et al. (2011)). Similar to social networks, we want to determine the key players in terrorist networks. Using game theoretic centrality measures rankings of players in such a terrorist network can be developed. The advantage of this game theoretic approach is that both the structure of the terrorist network as well as non-network features, such as financial means and bomb building skills, can be taken into account. Because game theoretic models are able to handle such additional information by assigning values to coalitions, this approach provides more realistic models to identify key players in a terrorist network.

In this paper we present how cooperative game theory can aid the identification of key players in a terrorist network. We introduce a weighted connectivity game that is able to take both the structure of the terrorist network as well as information about the individual terrorists into account. Applying a game theoretic centrality measure to the weighted connectivity game leads to a ranking of the players in the terrorist network. This allows for the optimal allocation of scarce observation resources and the destabilization of the terrorist network by the removal of the highest ranking members. To facilitate practical implementation of our methodology we present a general framework that includes three stages: construct the network, define the game theoretic model and analyze the rankings of players. We illustrate this framework by two practical cases: the Jemaah Islamiyah bombing in Bali and the 9/11 attack by Al Qaeda. The analyses of these cases with degree centrality, betweenness centrality and closeness centrality in concurrence with game theoretic connectivity centrality have led to some new results and insights. For example, in the case of Jemaah Islamiyah we identify one of the suicide bombers of the Bali bombing and in the case of Al Qaeda we identify a hijacker that links different airplanes that were used in the attack. We therefore state that quantitative centrality analyses provide a valuable contribution to the identification of key players in terrorist networks and henceforth are useful in combating the violent and disrupting phenomenon called terrorism.

This paper is organized as follows. After recapitulating the basic standard centrality measures in Section 2 we introduce a general framework for game theoretic centrality analysis in Section 3. We show how law enforcement and intelligence agencies can apply this framework to terrorist networks, in particular when additional information about the terrorist network is available. We also introduce the (weighted) connectivity game and a game theoretic centrality measure. In Section 4 we illustrate the practical use of centrality analyses in two case studies. We compare the standard centrality measures degree centrality, betweenness centrality and closeness centrality to case study specific game theoretic centrality measures. The cases that are discussed are the well known and documented Jemaah Islamiyah's Bali bombing and Al Qaeda's 9/11 attack.

2 Standard centrality

In this section we briefly recapitulate standard centrality, i.e., degree centrality, betweenness centrality and closeness centrality. A (social) network can mathematically be represented by a graph $g = (N, E)$, where the node set N represents the set of persons in the network and the set of edges E consists of all relationships that exist between these persons. A relationship

between person i and j is denoted by ij , with $ij \in E$.

The idea behind degree centrality is that the more people one knows the more important one is. The *normalized degree centrality* of person i is expressed as the fraction of the network to which person i is directly related:

$$C_{\text{degree}}(i) = \frac{d(i)}{|N| - 1}, \quad (1)$$

where $d(i)$ represents the number of direct relations of person i and $|N|$ is the total number of persons in the network. Observe that $0 \leq C_{\text{degree}}(i) \leq 1$.

Betweenness centrality was first introduced by Freeman (1977). The idea is that a person is important when he enables the flow of information between other persons in the network. Betweenness centrality is measured by counting the number of shortest paths (i.e., a path that uses a minimal number of links) between two persons that pass through another person. Let s_{kj} denote the total number of shortest paths between person k and j and let s_{kij} denote the number of shortest paths between k and j that pass through person i . The *normalized betweenness centrality* of person i is then defined by

$$C_{\text{between}}(i) = \frac{2}{(|N| - 1)(|N| - 2)} \cdot \sum_{\substack{k, j \in N \setminus \{i\} \\ k < j}} \frac{s_{kij}}{s_{kj}}, \quad (2)$$

from which it follows that $0 \leq C_{\text{between}}(i) \leq 1$.

Finally, closeness centrality quantifies the distance from a certain person to all other persons in the network. Borgatti and Everett (2006) argue that the essence of closeness centrality is *time-until-arrival* of entities that flow through a network, whereas betweenness centrality measures the *frequency-of-arrival* of flows in a network. The *normalized closeness centrality* of person i is defined by

$$C_{\text{close}}(i) = \frac{|N| - 1}{\sum_{j \in N} l_{ij}}, \quad (3)$$

where l_{ij} denotes the shortest distance between person i and j . Again, observe that $0 \leq C_{\text{close}}(i) \leq 1$.

Note that the actual standard centrality values are not important to us, only the resulting ordinal rankings of the persons involved are of interest. The following example illustrates the use of standard centrality.

Example: standard centrality measures

Consider the (social) network depicted in Figure 1. In this network the nodes represent 7 persons, denoted by the letters A to G , that are part of the network and the 10 links represent relationships between these 7 persons (note that a relationship is bidirectional).

Applying equations (1), (2) and (3), standard centrality can readily be computed for all persons in the network. Table 1 summarizes these results. Since only the ordinal rankings of the persons involved are of interest to us, these rankings are presented in Table 2. In this

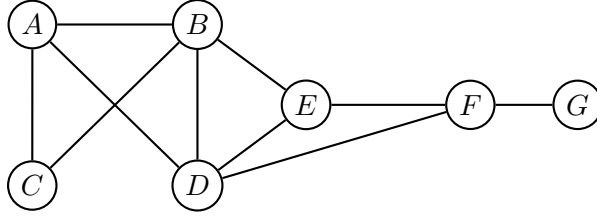


Figure 1: Example of a network.

latter table, the rankings are presented in descending order with an asterisk (*) and bullet (•) indicating equal rank.

It follows that different centrality measures yield different rankings of individuals, which is a direct result of the difference in context that these centrality measures try to capture. For example, in the network in Figure 1 person F has a lower degree centrality than person B . However, one can argue that importance not only depends on the number of persons one knows in a network, but also on the structural position in the network. Analyzing the structure of the network, person F seems to play an important role in keeping the network together. This observation is quantified by the betweenness centrality values of person F and person B ; i.e., $C_{\text{between}}(F) > C_{\text{between}}(B)$. In addition it can be seen that persons positioned at the periphery of the network score a betweenness centrality value of 0. For example, $C_{\text{between}}(C) = 0$. Note

| Person | Degree | Betweenness | Closeness |
|--------|--------|-------------|-----------|
| A | 0.5000 | 0.0778 | 0.6000 |
| B | 0.6667 | 0.2222 | 0.6667 |
| C | 0.3333 | 0 | 0.4615 |
| D | 0.6667 | 0.3222 | 0.7500 |
| E | 0.5000 | 0.1111 | 0.6667 |
| F | 0.5000 | 0.3333 | 0.6000 |
| G | 0.1667 | 0 | 0.4000 |

Table 1: Standard centrality for the network in Figure 1.

| Degree | Betweenness | Closeness |
|-------------|-------------|-------------|
| B^* | F | D |
| D^* | D | B^* |
| A^\bullet | B | E^* |
| E^\bullet | E | A^\bullet |
| F^\bullet | A | F^\bullet |
| C | C^* | C |
| G | G^* | G |

Table 2: Rankings for the network in Figure 1 based on standard centrality.

that although person A and person F rank equally high on degree centrality, only person F functions as a so-called gatekeeper in the sense that he connects different parts of the network. Considering closeness centrality, person D is the most important person in the network, due to his internal placement in the network. The three centrality measures are not in agreement about the key player in the network. Person B and person D , however, are ranked in the top 3 of each centrality measure. This indicates that both person B and person D play an important role in the network. □

Note that standard centrality only considers network *structure* and does not take additional information into account. Newman (2004) does consider weighted networks (i.e., a network where the relationships have assigned weights) by mapping a weighted network to its unweighted counterpart and applying standard centrality measures. However, this method still only analyzes the structure of the network and is not able to include information about (groups of) persons and their relationships. This motivates the need for a centrality measure that enables the use of such additional information.

3 Game theoretic centrality

In this section we introduce a game theoretic centrality measure to determine the key player in a terrorist network. Cooperative game theory studies situations in which players can generate benefits by working together. In this view a terrorist organization also consists of individuals that form (opportunity) coalitions in order to achieve a certain goal, e.g., to carry out an attack.

A cooperative game is a pair (N, v) , where N denotes the set of players. These players can cooperate and form different coalitions. A map v assigns a value $v(S)$ to each possible coalition $S \subseteq N$, which reflects the potential power coalition S represents. By definition $v(\emptyset) = 0$. We let the value for each possible coalition be defined by the network structure of the coalition as well as by additional information that is available for the coalition. To do so, we introduce a class of weighted connectivity games. It seems obvious to adopt a game that reflects the structural position of players in a terrorist network. Let the subgraph S_g consist of the players in coalition S and the lines of communication between these players. If the players in coalition S are able to communicate using only the relationships present within coalition S we say that subgraph S_g is connected and assign a value of 1 to coalition S . If not all players in coalition S are able to communicate then we say that subgraph S_g is not connected and we therefore assign a value of 0 to this coalition. A coalition consisting of a single person obtains a value of 0 by definition. Henceforth, the connectivity game v^{conn} (cf. Amer and Gimenez (2004)) is defined as

$$v^{\text{conn}}(S) = \begin{cases} 1 & \text{if } S_g \text{ is connected,} \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

Example: connectivity game

Consider again the network in Figure 1. The subgraph corresponding to, for example, coalition $\{D, E, F, G\}$ is connected, see Figure 2, whereas the subgraph corresponding to coalition $\{D, E, G\}$ is not, see Figure 3. Hence, $v^{\text{conn}}(\{D, E, F, G\}) = 1$ and $v^{\text{conn}}(\{D, E, G\}) = 0$. □

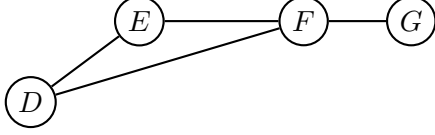


Figure 2: Subgraph for coalition $\{D, E, F, G\}$.

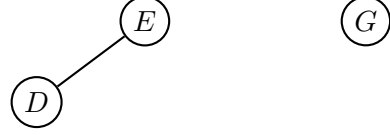


Figure 3: Subgraph for coalition $\{D, E, G\}$.

Besides the structural positions of the individuals in the terrorist network we would also like to model additional information that is available on these individuals and their relationships. For example, analysis of intelligence may show that a certain person has access to weapons, has financial means to set up an attack or attended a terrorist training camp. On the other hand, surveillance may show that the frequency of communication between two certain persons is much higher than the communication between other individuals in the network. Such additional information should be used to modify the value of coalitions in which these persons take part. A weighted connectivity game v^{wconn} enables the modeling of network structure as well as additional information. For example, in the case of Jemaah Islamiyah we define a weighted connectivity game that uses information about frequency and duration of communication between cell members and in the case of Al Qaeda we define a weighted connectivity game that uses personal information about hijackers, such as signs of radicalization and attendance at a terrorist training camp.

Next, we need to allocate the power of the coalition of all players in the network. The Shapley value (Shapley (1953)) can be viewed as the most prominent allocation rule in cooperative game theory. The Shapley value is based on the marginal contributions of a player to the different coalitions in order to measure the power of this player in the coalitions. Calculating a weighted average of these marginal contributions results in a ranking of the players in the network. A player, that on average contributes more than another player when added to a coalition, will have more power and thus will play a more important role in the network. This is reflected in the computation of the Shapley value $\varphi_i(v)$ of player i :

$$\varphi_i(v) = \sum_{S \subseteq N, i \notin S} \frac{|S|!(|N| - 1 - |S|)!}{|N|!} \cdot [v(S \cup \{i\}) - v(S)],$$

where $|S|$ is the number of players in coalition S . Players with a high Shapley value are ranked on top. In this paper the game theoretic centrality of person i is defined by

$$C_v(i) = \varphi_i(v). \quad (5)$$

Clearly, the weighted connectivity game used should reflect the problem at hand. Hence, in practice each new case leads to a new weighted connectivity game. To illustrate the definition of a weighted connectivity game and the ranking of players in a network we give two examples. The first example uses additional information about relationships between individuals. The second example uses personal information about individuals. Note that the corresponding weighted connectivity games only function to illustrate the use of additional information in game theoretic centrality. In section 4 an application of game theoretic centrality to two real-life cases is discussed.

Example: weighted connectivity game using information about relationships

Consider the network in Figure 1. Assume that surveillance results in additional information about relationship AC . For example, it is found that person A and person C communicate more frequently with one another than any other couple of individuals in the network. Therefore, the involved counterterrorism analyst assigns a weight of 4 to relationship AC , whereas all other relationships are assigned a weight of 1. Let the weighted connectivity game v^{wconn1} on the resulting weighted graph be defined as follows. The value of coalition S is equal to the maximum weight of the relationships that are present in coalition S if the underlying subgraph is connected and the value is equal to 0 in all other cases; i.e.,

$$v^{\text{wconn1}}(S) = \begin{cases} \max_{\substack{i,j \in S \\ i \neq j}} f_{ij} & \text{if } S_g \text{ is connected,} \\ 0 & \text{otherwise,} \end{cases} \quad (6)$$

where f_{ij} is the weight assigned to relationship ij in the network, with $f_{ij} = 0$ if $ij \notin E$. The choice for this specific game may be motivated by a terrorist cell structure in which there are many lines of communication and we need to focus on the most prominent lines in order to filter out the key players in the network. Using the weighted connectivity game v^{wconn1} in equation (6) the game theoretic centrality value $C_{v^{\text{wconn1}}}(i)$ can be computed for each player i . The resulting ranking of the players is presented in the column denoted by $W\text{conn1}$ in Table 3. To facilitate comparison this table also contains the previously obtained standard centrality results for the network (see Table 2).

| Degree | Betweenness | Closeness | Wconn1 |
|-------------|-------------|-------------|--------|
| B^* | F | D | A |
| D^* | D | B^* | C |
| A^\bullet | B | E^* | F |
| E^\bullet | E | A^\bullet | D |
| F^\bullet | A | F^\bullet | B |
| C | C^* | C | E |
| G | G^* | G | G |

Table 3: Rankings for the network in Figure 1 based on standard centrality and game theoretic centrality using information about relationships.

It can be seen from Table 3 that it is not immediately clear who the key player in the network is. Person B , D and F score high in most rankings, but only person F attains a top 3 ranking for all four centrality measures. Second, it can be seen that game theoretic centrality is better able to distinguish individuals than standard centrality. In other words, there are less persons of equal game theoretic rank than there are of standard centrality rank. This observation is strengthened by the results obtained in Section 4 for the cases of Jemaah Islamiyah and Al Qaeda. Third, the use of additional information affects the rankings of the persons concerned. In this example person A and person C are both ranked on top in the game theoretic centrality ranking. Fourth, comparing the rankings generated by standard centrality and game theoretic centrality leads to new insights in identifying the key players and the less important persons in a network. For

example, without additional information person C is found to play an insignificant role in the network. Using additional data person C scores higher on the game theoretic centrality ranking. This last observation is also supported by the results of our case studies in Section 4. \square

Example: weighted connectivity game using information about individuals

Now, assume that intelligence analysis shows that person E took part in a previous attack and that person C and person E both have sufficient financial means to their disposal to finance a potential attack. To model this additional information person C is assigned a weight of 4 and person E a weight of 11, whereas all other persons are assigned a weight of 1. The weighted connectivity game v^{wconn2} is defined as follows. The value of coalition S is equal to the sum of the weights of the players that are part of coalition S if the underlying subgraph is connected and the value is equal to 0 in all other cases; i.e.,

$$v^{\text{wconn2}}(S) = \begin{cases} \sum_{i \in S} w_i & \text{if } S_g \text{ is connected,} \\ 0 & \text{otherwise,} \end{cases} \quad (7)$$

where w_i is the weight assigned to person i in the network. The choice for this specific game may be motivated by the idea that the probability of launching a successful attack dramatically increases when experienced terrorists and terrorists with financial means or other useful skills team up. Using the weighted connectivity game v^{wconn2} in equation (7) the game theoretic centrality value $C_{v^{\text{wconn2}}}(i)$ can be computed for each player i . The resulting ranking of the players is presented in the column denoted by $W\text{conn2}$ in Table 4. This table also contains the previously obtained standard centrality results.

| Degree | Betweenness | Closeness | Wconn2 |
|-------------|-------------|-------------|--------|
| B^* | F | D | E |
| D^* | D | B^* | F |
| A^\bullet | B | E^* | B |
| E^\bullet | E | A^\bullet | D |
| F^\bullet | A | F^\bullet | C |
| C | C^* | C | A |
| G | G^* | G | G |

Table 4: Rankings for the network in Figure 1 based on standard and game theoretic centrality using information about individuals.

It can be seen from Table 4 that both person B and person F attain a top 3 ranking for all four centrality measures. Second, it can be seen that game theoretic centrality is again better able to distinguish individuals than standard centrality. Third, the use of additional information affects the ranking of person E and person C . This effect is more profound for person E , since he is ranked on top, whereas person C is only assigned a slightly higher position in the ranking when compared to standard centrality. Fourth, comparing the game theoretic centrality ranking with the standard centrality rankings leads to new insights. For example, comparing the positions

of person E in the various rankings calls for a further analysis of his role in the network. _____□

Identifying key players in a terrorist network by means of game theoretic centrality is based on both the structural position of each person as well as additional information about the individuals in the network and their relationships. In the previous two examples we presented two approaches to assign weights and construct a weighted connectivity game. Important steps in a practical implementation of our methodology consist of gathering information about the terrorist network, weighing the available data, defining a game theoretic model and analyzing the game theoretic and standard centrality rankings. In practice, these steps may overlap one another and the execution of the steps may follow a cyclic pattern. However, in general we can identify the following three stages: construct the network (input), define the game theoretic model (modeling) and analyze the rankings of players (output). We therefore propose to use the following framework.

General framework

The application of game theoretic centrality to a terrorist network includes the following three stages.

1. **Construct the network (input).** The object underlying a terrorist network is a weighted graph. The nodes of this graph represent the persons involved in the terrorist network and the links in the graph represent the relationships present between these persons. Furthermore, weights may be assigned to individuals and their relationships. The obtained data could originate from publicly available sources or from classified intelligence analysis. Experts in the counterterrorism and security domain are responsible for weighing each bit of information. When applying game theoretic centrality the weighted graph is considered to be a given input.
2. **Define the game theoretic model (modeling).** The behavior of persons in the terrorist network, how they interact and how they cooperate, has to be modeled in a weighted connectivity game. In practice, it is advisable to construct several weighted connectivity games in order to model a variety of possible scenarios. In the weighted connectivity game the value of each coalition of persons should be expressed in terms of the weights assigned to the persons and their relationships that are part of the coalition. Customization can be obtained by a suitable construction of the game. Therefore, both quantitative and intelligence experts play a crucial role in the construction of the game theoretic models.
3. **Analyze the rankings of players (output).** Using a game theoretic centrality measure, its value can be computed for the previously defined weighted connectivity game. In this way a ranking of persons in the terrorist network is obtained and key players can be identified. It is advised to compute standard centrality as well, since comparing the rankings obtained by standard and game theoretic centrality may lead to new insights in identifying the most important players in a terrorist network.

Note that the game used in our framework depends on the context of the problem at hand. The game should reflect the behavior of the persons in the terrorist network, thereby making full

use of any additional information available about individuals and their relationships. In other words, each new terrorist network calls for a new game. In the next section the above framework is used to determine the key players for the cases of the Jemaah Islamiyah Bali bombing and Al Qaeda's 9/11 attack.

4 The cases of Jemaah Islamiyah and Al Qaeda

In this section we illustrate the application of game theoretic centrality to two terrorist networks: the operational network of Jemaah Islamiyah's Bali bombing and the network of hijackers of Al Qaeda's 9/11 attack. For each of these two networks we define a case specific weighted connectivity game that incorporates additional information about the individuals and relationships involved.

4.1 Case 1: Jemaah Islamiyah in Bali

On October 12th, 2002, one of the deadliest attacks in Indonesia's history took place on the island of Bali. In total 202 innocent civilians died as a result of this attack. After a long trial a number of members of the violent extremist group Jemaah Islamiyah were found guilty of planning and perpetrating this attack.

Jemaah Islamiyah was officially founded in 1993 in Malaysia. Its goal became the founding of an Islamic state in Indonesia (Wise (2005)). In 1998 Jemaah Islamiyah started the so-called *uhud* project. The aim of this project was to remove Christians as well as Hindus from regions in Indonesia, such that pure Islamic enclaves could be founded that were guided by Sharia-law (Abuza (2003)). In addition Jemaah Islamiyah started a series of attacks in 2000. The 2002 Bali attack being its most prominent one.

The tactical operation in Bali was conducted by Jemaah Islamiyah's Indonesian cell, headed by Hambali. A suicide terrorist detonated an explosive vest in Paddy's bar. This caused many people to flee into the streets. A second explosion followed, caused by a so-called vehicle born improvised explosive (VBIED): an L300 van filled with about 1000 kilograms of TNT and ammonium nitrate. These explosions resulted in the death of 202 people.

In order to apply a game theoretic centrality analysis to the Bali attack we need information about the operational cell that was responsible for this attack. We have used data available from scientific literature, in particular data from a social network analysis of Jemaah Islamiyah by Koschade (2006). In his paper Koschade not only defines the network structure, but also assigns weights to relationships between cell members. These weights, however, are not used in his analysis of the operational network. We show how game theoretic centrality can incorporate this additional information in its analysis. Our analysis follows the three stages of the framework in Section 3.

1. **Construct the network.** We consider the operational network of the Bali attack as presented by Koschade (2006). The corresponding network is depicted in Figure 4. The operational cell conducting the attack consisted of three teams: a team of bomb builders (gray), a support team (lightgray) and a team responsible for coordinating the attack (white). The team of bomb builders consisted of Patek, Ali Imron, Azahari, Dulmatin, Ghoni, Sarijo and later on Feri was added to this team. The persons responsible for the

support of the operation (team Lima) were Octavia, Junaedi, Hidayat, Abdul Rauf and Arnasan. The remaining persons, i.e., Samudra, Idris, Muklas, Amrozi and Mubarak, were in charge of coordinating the attack. These 17 cell members and their 63 relationships define the structure of the operational network. To determine the strength of existing relationships in the network, interactions between the 17 cell members were recorded from October 6 to October 11. Weighing these recordings using the criteria *transactional content* and *frequency and duration* of interaction, each interaction was assigned a weight between 0 and 5 (Koschade (2006)). A weight of 0 implies there is no relationship at all between two individuals (based on the recordings), whereas a weight of 5 means that there is highly frequent interaction of long duration between cell members. In Figure 4 these weights are visualized by the thickness of the lines connecting the cell members; i.e., the thicker the line the higher the weight assigned to the corresponding relationship.

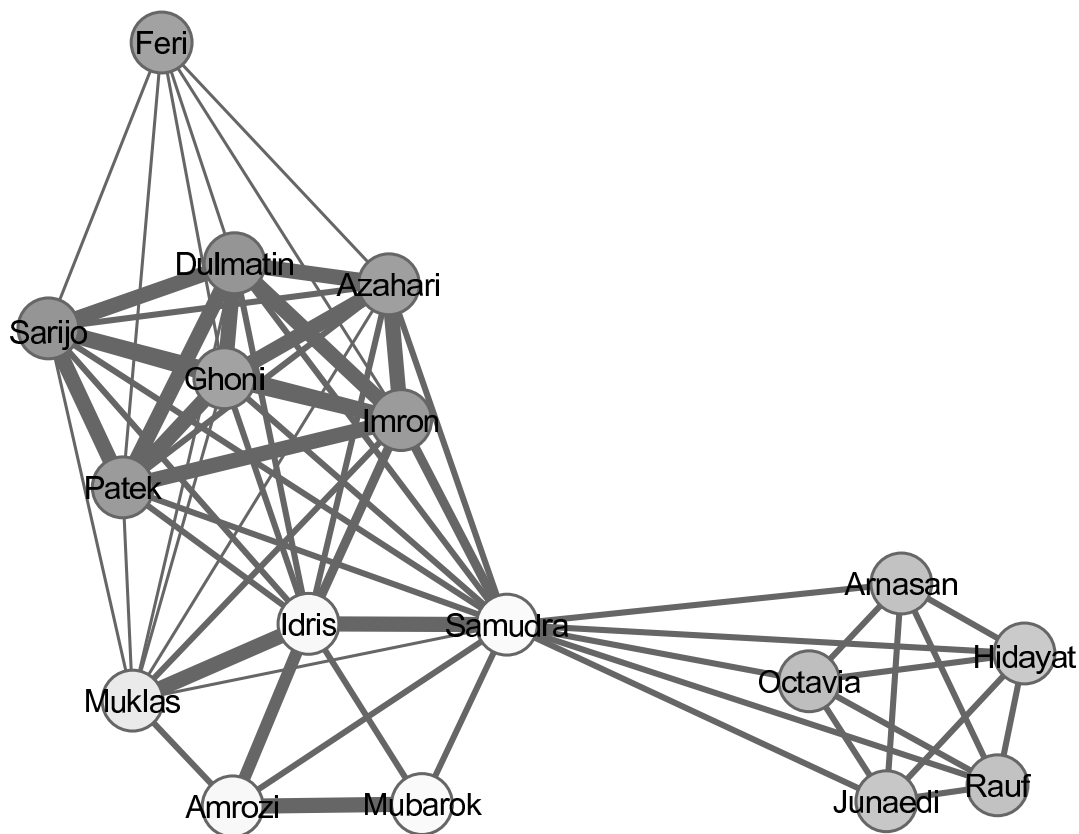


Figure 4: Operational network of Jemaah Islamiyah's Bali attack. Coordination team (white), support team (lightgray) and bomb building team (gray).

2. **Define the game theoretic model.** At this stage we need to define a weighted connectivity game that uses the available data and reflects the context of the problem at hand; i.e., a terrorist cell that tries to prevent discovery during the planning and execution phase of an attack. Therefore, we motivate our choice of the case specific weighted connectivity game as follows. A terrorist organization will try to shield its important players by keep-

ing the frequency and duration of their interaction with others to a minimum. However, to be able to coordinate and control the attack an important player needs to maintain relationships with other individuals in the network. The power of a coalition is therefore defined by the total number of relationships that exist within that coalition divided by the sum of the weights (representing frequency and duration of interaction) on those relationships; i.e.,

$$v^{\text{wconn}}(S) = \begin{cases} \frac{\sum_{i,j \in S, i \neq j} I_{ij}}{\sum_{i,j \in S, i \neq j} f_{ij}} & \text{if } S_g \text{ is connected,} \\ 0 & \text{otherwise,} \end{cases} \quad (8)$$

where f_{ij} is the weight assigned to relationship ij in the network, with $f_{ij} = 0$ if $ij \notin E$. If relationship ij is present in the network (i.e., if $ij \in E$) then $I_{ij} = 1$, else $I_{ij} = 0$. To clarify the idea behind equation (8), consider all coalitions with a low frequency and duration of interaction. If a certain player facilitates communication between individuals in many such coalitions (i.e., he makes the underlying subgraphs connected) then this player will attain a high ranking. Note that if $f_{ij} = 1$ for all relationships $ij \in E$ then equation (8) equals the definition of the connectivity game v^{conn} in equation (4). It can thus be seen that the weighted connectivity game not only takes the structure of the terrorist network into account but it also models additional information that is available on the relationships between cell members.

3. **Analyze the rankings of players.** The game theoretic centrality value of each cell member can be computed for the game v^{wconn} in equation (8). The resulting rankings for standard as well as game theoretic centrality are presented in Table 5. In this table ranking is presented in descending order with an asterisk (*) and bullet (•) indicating equal rank.

In practice capacity for surveillance is limited. We therefore focus on the top 5 of highest ranked cell members at each centrality measure, as indicated by the bars in Table 5. Note that there may be more than 5 persons in the top 5 due to the fact that some individuals attain equal centrality values. Analyzing the different rankings leads to the following observations. First, we conclude that Samudra was the key player in this operation. Each centrality measure defines him as the most important person in the operational network. Second, if only network structure is taken into account, the rankings of the 5 most important persons are ambiguous. With capacity of surveillance limited to a total of 5 individuals, it is not clear which persons to watch more closely. Only game theoretic centrality is able to distinguish between the top ranking players in the network. Third, we observe that the game theoretic ranking introduces three new persons in the top 5, when compared to standard centrality; i.e., Feri, Azahari and Sarijo attain a top 5 ranking at the expense of Idris, Imron and Dulmatin. In particular, the way Feri advances from an insignificant position in the standard centrality rankings to a third place in the game theoretic ranking calls for a further analysis of his role in the operational network.

It is safe to say that Samudra's removal from the network would have had a pronounced effect

| Degree | Betweenness | Closeness | Wconn |
|-------------------------|-------------------------|-------------------------|-------------|
| Samudra | Samudra | Samudra | Samudra |
| Idris | Idris | Idris | Muklas |
| Muklas* | Muklas | Muklas* | Feri |
| Ali Imron* | Ali Imron* | Ali Imron* | Azahari |
| Dulmatin* | Dulmatin* | Dulmatin* | Sarijo |
| Azahari* | Azahari* | Azahari* | Patek |
| Patek* | Patek* | Patek* | Dulmatin |
| Ghoni* | Ghoni* | Ghoni* | Idris |
| Sarijo* | Sarijo* | Sarijo* | Ghoni |
| Feri | Amrozi | Arnasan [•] | Octavia* |
| Arnasan [•] | Feri [•] | Junaedi [•] | Abdul Rauf* |
| Junaedi [•] | Arnasan [•] | Abdul Rauf [•] | Hidayat* |
| Abdul Rauf [•] | Junaedi [•] | Octavia [•] | Arnasan* |
| Octavia [•] | Abdul Rauf [•] | Hidayat [•] | Junaedi* |
| Hidayat [•] | Octavia [•] | Amrozi | Amrozi |
| Amrozi | Hidayat [•] | Mubarok | Mubarok |
| Mubarok | Mubarok [•] | Feri | Ali Imron |

Table 5: Rankings for the Jemaah Islamiyah network based on standard and game theoretic centrality.

on the Bali operation. This is in concordance with a ruling by judge Sudewi (The New Zealand Herald (2003)):

“Judge Isa Sudewi told the court today the prosecution had proven Samudra, an engineering graduate, played a key role in the bombings. ‘The defendant worked behind the scenes as the coordinator so the panel of judges has an opinion that the defendant is the intellectual actor behind the bomb explosions,’ she said.”

From our analysis it follows that if additional information is added to the network structure, other persons turn up as high ranking cell members. Feri, for example, conducted an important task during the Bali bombing. He arrived on October 10th, two days prior to the attack, and was recruited to be the suicide bomber of Paddy’s bar. Another cell member that is identified by game theoretic centrality is Azahari. He was Jemaah Islamiyah’s bomb expert and is considered to have been the ‘brain’ behind the Bali operation (Council on Foreign Relations (2009)). If Feri’s or Azahari’s role would have been detected or recognized in time the feasibility of the operation would have been seriously hampered. It thus follows that taking additional information into account (in this case study weights on the relationships between cell members) the results of a game theoretic centrality analysis can lead to new insights in a terrorist network. As a result, surveillance can more effectively be allocated due to the increased ability to distinguish between the importance of different individuals.

4.2 Case 2: Al Qaeda and 9/11

On Tuesday morning (local time) September 11th, 2001, the world was shocked by two planes flying into the Twin Towers of the World Trade Center in New York. A third plane flew into the Pentagon and a fourth plane crashed somewhere in Pennsylvania. It turned out that 19 hijackers, most of whom were from Saudi Arabia, were directly responsible for the execution of the operation. The events leading up to this day have been described meticulously in popular media and the academic literature, e.g., Kean et al. (2002).

Three and a half years before the infamous 9/11 attack Osama Bin Laden issued a *fatwa* calling on all Muslims “to kill the Americans, both civilian and military, in every country in which it was possible to do so...” (Al Quds Al Arabi (1998)). Already in 1996 he issued a *declaration of jihad* against the United States (Al Islah (1996)). Furthermore, Bin Laden expressed his wish that the United States would withdraw from Saudi Arabia. He argued that the presence of American troops on the Arabian peninsula was an insult to the Islamic community.

During a presentation in Tora Bora, Khalid Sheikh Mohammed proposed an operation with trained pilots flying into buildings (Kean et al. (2002)). This proposal finally culminated in the 9/11 attack. Note that Khalid Sheikh Mohammed was also in contact with Hambali, Jemaah Islamiyah’s Indonesian cell leader. During the summer and autumn of 2000 the hijackers were selected by Bin Laden and his followers. These hijackers arrived in the United States in April 2001. The specific date of September 11th was probably only determined somewhere in August 2001. Several days before the actual attack the hijackers relocated to hotels close to their designated airports and the remaining finances were transferred.

The data we use in our game theoretic centrality analysis of the 9/11 attack originates from two publicly available sources; i.e., a social network analysis of the network of hijackers by Krebs (2002) and the 9/11 commission report by Kean et al. (2002). Again, we follow the three stages of the framework in Section 3.

1. **Construct the network.** To conduct a centrality analysis of the network responsible for the 9/11 operation we used network data that were gathered by Krebs (2002). He obtained data on the hijackers from open sources, like major newspapers. The network corresponding to our data is depicted in Figure 5. The colors in the network refer to the different flights of American Airlines (AA) and United Airlines (UA); i.e., AA-77 (white), AA-11 (lightgray), UA-93 (gray) and UA-175 (darkgray).

The power of game theoretic centrality analysis is the ability to incorporate additional information in the analysis of a terrorist network. There are many ways to obtain such additional information. For example, in the case study of Jemaah Islamiyah’s Bali attack we used additional information that reflected the frequency and duration of interaction between cell members. A first look at the publicly available data on Al Qaeda’s 9/11 attack shows that it only contains binary network information. Reports on the 9/11 attack, however, reveal more information on the individuals that perpetrated this attack. Using the 9/11 commission report of Kean et al. (2002) we were able to obtain additional information on some of the hijackers, which we characterized under indicators like affiliation and signs of radicalization. Table 6 presents the additional data extracted from the 9/11

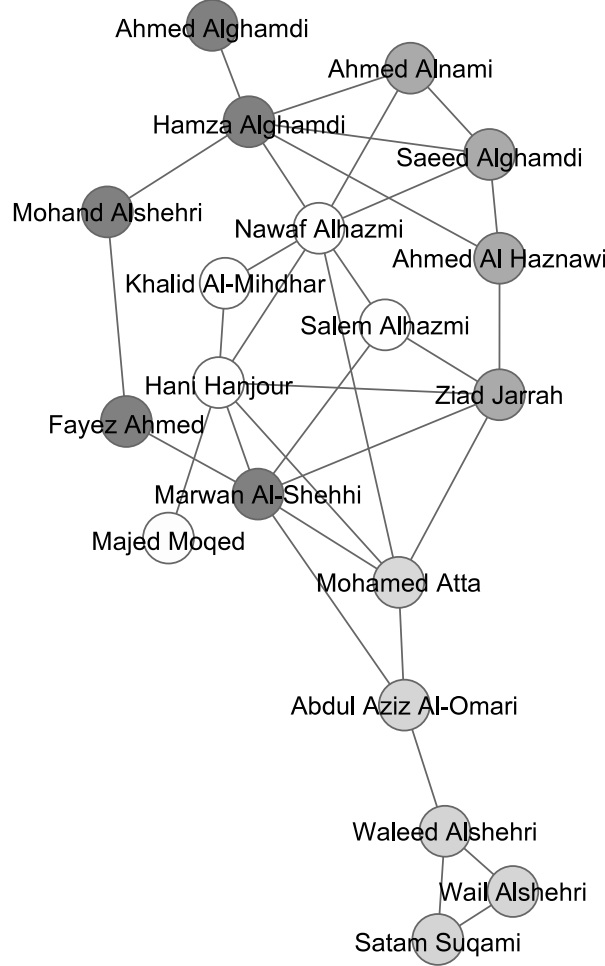


Figure 5: Operational network of hijackers of Al Qaeda's 9/11 attack. AA-77 (white), AA-11 (lightgray), UA-93 (gray) and UA-175 (darkgray).

commission report. Note that in general a thorough analysis of historical sources will shed more light on the persons involved in an attack.

We determine the weight w_i of each person i as follows. First, every person in the network is assigned a weight of 1. Next, for each person it is determined which indicators are relevant and the weight assigned to this person is increased accordingly. As Table 6 indicates, we have chosen to let each relevant indicator increase the person's weight by 1. For example, Mohamed Atta is assigned a weight of 4, whereas Majed Moqed is assigned a weight of only 1. Table 7 presents the total weight of each hijacker when considering the indicators defined in Table 6.

2. **Define the game theoretic model.** Given the network of hijackers and the weights assigned to each hijacker we want to identify the key players. Therefore, we need to define a weighted connectivity game that not only uses the structure of the network but also incorporates these available data. We motivate our choice of the case specific weighted

| Description indicator | Example(s) | Person(s) | Weight |
|--|--|---|--------|
| Attending meetings on terror attack planning | Kuala Lumpur meeting January 2000 | Nawaf Al-Hazmi Khalid Al-Midhar | +1 |
| Signs of radicalization | Antisemitic and anti-American speech, talk about jihad and martyrdom, writing a will | Mohamed Atta Marwan Al-Shehhi Ziad Jarrah | +1 |
| Affiliations | Al-Quds mosque Hamburg | Mohamed Atta Ziad Jarrah | +1 |
| Accomplice to previous attacks | Attack on USS Cole | Khalid Al-Midhar | +1 |
| Attending terrorist training camps | Traveling to training camps in Pakistan and Afghanistan | Mohamed Atta Marwan Al-Shehhi Ziad Jarrah | +1 |

Table 6: Example of some indicators and assigned weights.

| Person | Total weight | Person | Total weight |
|------------------|--------------|---------------------|--------------|
| Ahmed Alghamdi | 1 | Nawaf Alhazmi | 2 |
| Hamza Alghamdi | 1 | Khalid Al-Mihdhar | 3 |
| Mohand Alshehri | 1 | Hani Hanjour | 1 |
| Fayez Ahmed | 1 | Majed Moqed | 1 |
| Marwan Al-Shehhi | 3 | Mohamed Atta | 4 |
| Ahmed Alnami | 1 | Abdul Aziz Al-Omari | 1 |
| Saeed Alghamdi | 1 | Waleed Alshehri | 1 |
| Ahmed Al-Haznawi | 1 | Satam Suqami | 1 |
| Ziad Jarrah | 4 | Wail Alshehri | 1 |
| Salem Alhazmi | 1 | | |

Table 7: Weights assigned to hijackers of Al Qaeda’s 9/11 attack.

connectivity game as follows. Individuals that score high on the indicators defined in Table 6 play an important part in the operation. When such individuals team up, they have a significant effect on the potential success of the operation. We therefore define the value of a coalition to be the sum of the weights of its players; i.e.,

$$v^{\text{wconn}}(S) = \begin{cases} \sum_{i \in S} w_i & \text{if } S_g \text{ connected,} \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

For example, the value of the coalition consisting of Mohamed Atta, Marwan Al-Shehhi and Abdul Aziz Al-Omari equals $4 + 3 + 1 = 8$, whereas the coalition consisting of Mohamed Atta, Marwan Al-Shehhi and Waleed Alshehri equals 0 since not all players in this latter coalition can communicate, see Figure 5. It can thus be seen that the weighted

connectivity game is able to model both network structure and weights assigned to the hijackers.

3. **Analyze the rankings of players.** The game theoretic centrality value of each hijacker can be computed for the game v^{wconn} in equation (9). The resulting rankings for standard as well as game theoretic centrality are presented in Table 8. In this table ranking is presented in descending order with players that attain equal rank indicated by the symbols *, •, ◊, ★ and ◦.

| Degree | Betweenness | Closeness | Wconn |
|-------------------|------------------|------------------|------------------|
| N. Alhazmi | N. Alhazmi | N. Alhazmi* | A. Aziz Al-Omari |
| M. Al-Shehhi* | A. Aziz Al-Omari | M. Atta* | H. Alghamdi |
| H. Alghamdi* | M. Atta | M. Al-Shehhi• | Wd. Alshehri |
| H. Hanjour* | M. Al-Shehhi | H. Hanjour• | H. Hanjour |
| M. Atta• | Wd. Alshehri | Z. Jarrah | M. Al-Shehhi |
| Z. Jarrah• | H. Alghamdi | H. Alghamdi◊ | M. Atta |
| S. Alghamdi | H. Hanjour | S. Alhazmi◊ | N. Alhazmi |
| A. Aziz Al-Omari◊ | Z. Jarrah | A. Aziz Al-Omari | Z. Jarrah |
| Wd. Alshehri◊ | F. Ahmed | S. Alghamdi | M. Alshehri |
| A. Al-Haznawi◊ | M. Alshehri | A. Al-Haznawi | K. Al-Midhar |
| S. Alhazmi◊ | A. Al-Haznawi | F. Ahmed* | A. Al-Haznawi |
| A. Alnami◊ | S. Alhazmi | A. Alnami* | F. Ahmed |
| F. Ahmed* | S. Alghamdi* | K. Al-Midhar | S. Alhazmi |
| M. Alshehri* | A. Alnami* | M. Alshehri | S. Alghamdi |
| K. Al-Midhar* | K. Al-Midhar* | M. Moqed | A. Alnami |
| S. Suqami* | S. Suqami* | Wd. Alshehri | S. Suqami* |
| W. Alshehri* | W. Alshehri* | A. Alghamdi | W. Alshehri* |
| A. Alghamdi◊ | A. Alghamdi* | W. Alshehri◊ | A. Alghamdi |
| M. Moqed◊ | M. Moqed* | S. Suqami◊ | M. Moqed |

Table 8: Rankings for Al Qaeda’s 9/11 network based on standard and game theoretic centrality.

We again focus on the top 5 of highest ranked individuals at each centrality measure, as indicated by the bars in Table 8. Analyzing the different rankings leads to the following observations. First, we conclude that standard and game theoretic centrality identify a different key player; i.e. Nawaf Alhazmi versus Abdul Aziz Al-Omari. The destabilizing effect to the network after the removal of Nawaf Alhazmi is only minor. Since Abdul Aziz Al-Omari forms a *bridge* between two parts of the network the destabilizing effect of his removal is more profound, see Figure 5. Second, there are other hijackers that form bridges in the network, for instance Hamza Alghamdi. Together with Marwan Al-Shehhi he links United Airlines flight 175 with the remaining hijackers. Additionally it can be seen that Abdul Aziz Al-Omari and Waleed Alshehri are essential in linking American Airlines flight 11 to the remainder of the network. This is why these hijackers are ranked high according to game theoretic centrality. Third, when considering only network structure the rankings of hijackers are ambiguous. Only game theoretic centrality is able to distinguish between

all hijackers (with the exception of the two symmetric players Satam Suqami and Wail Alshehri).

The difference between the suggested key players Nawaf Alhazmi and Abdul Aziz Al-Omari calls for a further analysis of their role in the network of hijackers. Nawaf Alhazmi was known to be regularly in contact with Mohamed Atta during the summer of 2001. It is assumed that the latter conducted an important role in planning the operational part of the attack (Los Angeles Times (2002)). Abdul Aziz Al-Omari, together with Mohamed Atta, constituted the link between the hijackers of American Airlines flight 11 and the remainder of the network. Using only a marginal amount of additional information has led to more insight in the role of and the relationship between Nawaf Alhazmi and Abdul Aziz Al-Omari and the destabilizing effect that would be invoked by the removal of these hijackers from the network responsible for the 9/11 attack. Hence, even in case of limited additional information we can profit from the comparison between standard centrality and game theoretic centrality.

5 Conclusions

In this paper we introduced a quantitative methodology to identify key players in terrorist networks. Our methodology combines solution concepts from cooperative game theory with social network analysis. The advantage of our game theoretic approach is that both the structure of a terrorist network as well as non-network features, such as money and bomb building skills, can be taken into account. Since game theoretic models are able to handle such additional information our approach provides more realistic models to identify key players. We illustrated our methodology by two case studies: Jemaah Islamiyah's Bali bombing and Al Qaeda's 9/11 attack. This has led to new insights in the operational networks responsible for these attacks.

Centrality measures can be used to construct rankings of important players in terrorist networks. In the case studies of Jemaah Islamiyah and Al Qaeda we applied both standard centrality, i.e., degree centrality, betweenness centrality and closeness centrality, and game theoretic case specific centrality measures to construct rankings of the terrorists involved. Our analyses included three stages: construct the network (input), define the game theoretic model (modeling) and analyze the rankings of players (output). The characteristics of each centrality measure, following from the results of our case studies, are presented in Table 9.

Table 9 should be interpreted as follows. Game theoretic centrality, for example, considers all possible coalitions of terrorists in a given network (*network structure*). Furthermore, personal information about terrorists and their interrelationships can be taken into account (*additional information*). Finally, the centrality measure is able to distinguish individuals when ranking terrorists in order of their importance to the network (*level of differentiation*).

Game theoretic centrality contributes to social network analysis by facilitating the incorporation of additional information when constructing rankings of the players in a terrorist network. This leads to more realistic rankings and, when compared to standard centrality rankings, may lead to new insights in the terrorist network under consideration. Both these conclusions are validated by the case studies of Jemaah Islamiyah and Al Qaeda.

Note that all data used in this paper originate from publicly available resources. It is to be expected that the quality of quantitative analyses of terrorists networks increases when more accurate and trustworthy (classified) data are available. Nevertheless, using only publicly

| Centrality measure | Network structure | Additional information | Level of differentiation |
|--------------------|--------------------------------|------------------------|--------------------------|
| Degree | Direct relationships | — | — |
| Betweenness | Connection between individuals | — | + |
| Closeness | Distance between individuals | — | + |
| Game theoretic | Coalitions of individuals | + | ++ |

Table 9: Characteristics of standard and game theoretic centrality measures.

available data sets, game theoretic centrality analysis provides valuable new insights in the operational networks of Jemaah Islamiyah and Al Qaeda.

Finally, this paper focuses on constructing rankings of players in a terrorist network. When key players have been identified, the next step is to determine how to act in order to maximize damage to the terrorist organization. For example, does a key player need to be put under extra surveillance in order to gain more information about the organization as a whole or should this player be eliminated from the network in order to destabilize the lines of communication? To answer this question further quantitative analysis of the network is needed.

Acknowledgement

The authors thank the members of a special project group of the Research and Documentation Centre (WODC) of the Dutch Ministry of Security and Justice and an advisor of the National Coordinator for Counterterrorism (NCTb) for their valuable comments on an earlier version of this manuscript.

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